vealed that the momentum thickness responds in a linear manner to the passing wakes, but the skin friction distribution exhibits significant nonlinear behavior.

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Estimation of Mixing of High-Speed Streams

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Nomenclature

D = diameter of mixing tube
E_Q = basic unit based on Q
G = mass flow flux
K_a = factor of uniformity of total pressure between axes
K_r = factor of radial uniformity of total pressure
M = Mach number
MF = momentum flux
P = total pressure
p = static pressure
Q = physical quantity
r = radial coordinate
v = velocity
x = axial coordinate
γ = ratio of specific heats
η = mixing parameter
ρ = density

Subscripts

A, B = measurement locations in a plane perpendicular to the flow direction
m = mean value along radial direction

Introduction

ESTIMATION of experimental mixing performance is critical to assess the various passive supersonic mixing enhancement devices[1-3] and ejector systems.[4,5] The problem gets further compounded in the mixing of flows with the same inlet total temperature, and using the same working medium, as stream identification is difficult. Decay of centerline properties and spreading rates of jets have been used by many investigators[1,3] as indicators of mixing performance. However, as noted by Naughton et al.[6] these parameters are more representative of the penetration of the jet into the surrounding medium. Mixing parameters have been based on momentum flux,[2,5] as it remains constant across shocks (from inviscid, nonfrictional one-dimensional flow analysis). However, a parametric study is not available to justify this choice. Though uniformity of mean flow profiles has been used by many investigators to evaluate mixing performance, Dimotakis[7] suggested that an increasingly uniform distribution of mean flow quantities is not the ultimate check for molecular scale mixing essential for combustion. However, gas sampling measurements of Gutmark et al.[8] suggest that pressure measurements can be used to make reasonable estimates of mixing rates.

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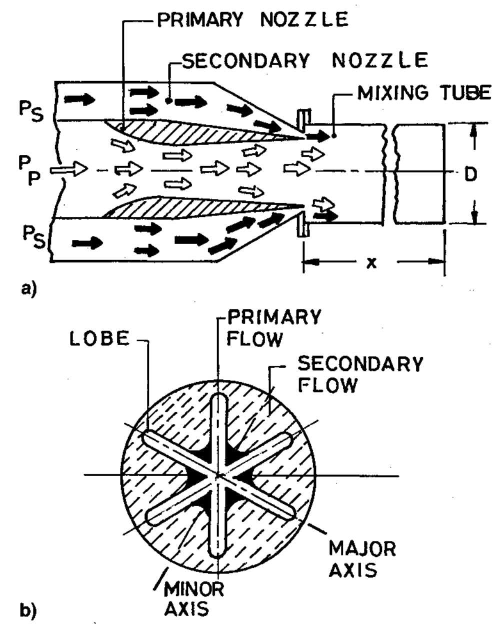

Fig. 1 Flow configuration: a) schematic and b) end view of a radially lobed nozzle.

This Note suggests a simple method to obtain a physically meaningful estimate of mixing performance in the context of interacting coaxial streams (see Fig. 1a) using typical discrete experimental data. As efficient mixing of the coaxial streams with different inlet conditions involves exchange of properties until the gradients are reduced, the emphasis is on a parameter that amplifies any existing difference (to highlight the extent of the lack of mixing) in physical quantities in a given flow cross section.

The extent of mixing of flows is assessed at planes perpendicular to the mean flow direction. First the basic unit used in η is developed; later η is formulated for an axisymmetric and a nonaxisymmetric flow.

Results and Discussion

Formulation of Basic Unit

If the basic unit E_Q of η uses a physical quantity, the difference $Q_B - Q_A$ should reduce as the flow becomes increasingly mixed. Because of the dissipative nature of efficient mixing devices, normalization of $(Q_B - Q_A)$ in the basic unit is done with local quantities. Because of its greater amplification of the difference, Q_A is preferred over the average of Q_A and Q_B as the normalizing factor. To incorporate the effects of pressure, density, and velocity on the mixing process, the physical quantities considered were 1) momentum flux $p + \rho v^2$, 2) mass flow flux ρv, and 3) total pressure. Again, the physical quantity that yields the largest value for the basic unit is chosen.

Based on the measurements at locations A and B, without loss of generality, it can be assumed that $p_B = p_A + \Delta p$ and

$M_B = M_A + \Delta M$, locations A and B being chosen such that $\Delta M \geq 0$.

Assumptions used in the mean flow analysis to obtain the most appropriate physical quantity are 1) ideal gas equation is valid, 2) at any particular location the static and total properties are related by isentropic equations, and 3) for simplicity of analysis and to obtain a parameter specifically suitable for flow regions that are reasonably mixed, the restriction that $|\Delta M/M_A|$ and $|\Delta p/p_A|$ are smaller than unity is imposed. However, it can be seen that assumption 3 could be relaxed without significantly altering the validity of the final result.

Formulating the basic unit E_{MF} with momentum flux:

$$\mathrm{MF}_A = p_A(1 + \gamma M_A^2) \quad (\gamma \text{ is assumed to remain constant})$$

$$\mathrm{MF}_B = (p_A + \Delta p)[1 + \gamma(M_A + \Delta M)^2]$$

Simplification, considering only first-order terms, yields

$$\mathrm{MF}_B = \mathrm{MF}_A\left(1 + \frac{\Delta p}{p_A} + \frac{2\gamma M_A \Delta M}{1 + \gamma M_A^2}\right) \Rightarrow E_{\mathrm{MF}}$$

$$E_{\mathrm{MF}} = \frac{\mathrm{MF}_B - \mathrm{MF}_A}{\mathrm{MF}_A} = \frac{\Delta p}{p_A} + \frac{2\gamma M_A \Delta M}{1 + \gamma M_A^2} \tag{1}$$

Proceeding on similar lines

$$\Rightarrow E_G = \frac{\Delta p}{p_A} + \frac{\Delta M}{M_A} + \frac{(\gamma - 1)M_A \Delta M}{2 + (\gamma - 1)M_A^2} \tag{2}$$

$$\Rightarrow E_P = \frac{\Delta p}{p_A} + \frac{\gamma M_A \Delta M}{1 + [(\gamma - 1)/2]M_A^2} \tag{3}$$

After algebraic manipulation it can be seen that $E_P \geq E_{\mathrm{MF}} \geq E_G$ for supersonic Mach numbers. Hence, total pressure is used in the formulation of the basic unit and is of the form $E_P = (P_B - P_A)/P_A$ (for $M_B \geq M_A$).

Computation of exact values of E_P, E_{MF}, and E_G for several cases confirmed the results of the previous analysis. The result was also found to be valid for combinations of values that violated assumption 3 (e.g., for $p_A = 0.08$ MPa, $\Delta p/p_A = 2.0$, and $\Delta M/M_A = 3.0$, it was found that E_P was considerably larger than E_{MF} and E_G for supersonic Mach numbers).

Formulation of η

The form of the basic unit developed was used to estimate the mixing of coaxial high-speed streams in a confining tube (Fig. 1a). The mixing parameter η was formulated for 1) a conical (axisymmetric) and 2) a radially lobed (nonaxisymmetric, Fig. 1b) primary nozzle. Further details about the nozzles can be found in Ramesh Kumar and Kurian.[9] Radial distribution of total pressure at the exit plane of the mixing tube at a given axial location was used to compute η (the pressure distribution for the lobed nozzle was obtained along the major and minor axes).

In experiments using a conical primary nozzle, nonuniformity of flow conditions exists only in the radial direction necessitating a check for uniformity:

$$K_r^2 = \frac{1}{R}\int_0^R [E_P(r)]^2 \, \mathrm{d}r$$

where $R = D/2$ and $E_P(r) = [P(r) - P_m]/P_m$ {if $M(r) \geq M_m$ or vice versa, i.e., $E_P(r) = [P_m - P(r)]/P(r)$}.

When a lobed primary nozzle is used, the flow conditions differ along the radial directions in the minor and major axes. Hence,

$$K_r = (K_{r\mathrm{major}} + K_{r\mathrm{minor}})/2$$

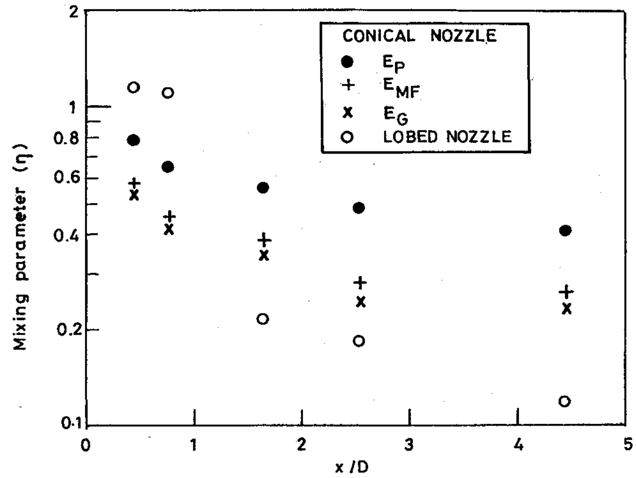

Fig. 2 Variation of the mixing parameter with axial location for the nozzles ($M_{\mathrm{Primary}} = 1.7$ and $M_{\mathrm{Secondary}} = 1.0$).

As dissimilarity of flow properties in the major and minor axes induces transport of properties until the gradients are reduced, an additional check for uniformity between the axes is necessary for the nonaxisymmetric nozzle.

Hence, the following equation was obtained:

$$\frac{1}{\pi R^2}\int_0^R [E_P(r)]^2 2\pi r \, \mathrm{d}r = K_a^2$$

where $E_P(r) = [P(r)_{\mathrm{major}} - P(r)_{\mathrm{minor}}]/P(r)_{\mathrm{minor}}$ [if $M(r)_{\mathrm{major}} \geq M(r)_{\mathrm{minor}}$ or vice versa].

Thus,

$$\begin{aligned} \eta &= K_r && \text{for conical nozzle} \\ \eta &= (K_r + K_a)/2 && \text{for lobed nozzle} \end{aligned} \tag{4}$$

and the lower the values of η the better the mixing of the flows.

Figure 2 shows the variation of η for the conical and lobed nozzles at several axial locations x. Also shown for the conical nozzle is η based on momentum flux and mass flow flux. This figure clearly shows that η based on total pressure gives a more conservative estimate of the mixing performance and the largest range of variation with axial distance. A similar trend was also observed for the lobed nozzle. Because of the nonaxisymmetric nature of flow, the value of η for the lobed nozzle is high initially, and low at farther downstream locations because of the superior mixing performance of the lobed nozzle. Computation of the pressure drop characteristics of the nozzles using the method of Fuller et al.[10] showed a similar trend. This was anticipated, as mixing is inevitably associated with pressure drop. Thus, η is also found to reflect the pressure drop characteristics of the flow.

In conclusion, total pressure is the more appropriate physical quantity to be used to check the extent of mixing. Further, the mixing parameter proposed is easy to evaluate and found to be valid over a range of real flow conditions, and can be readily adapted to other flow configurations. The final formulation of the mixing parameter could be easily extended to incorporate a larger set of measurements for nonaxisymmetric nozzles and could also be applied to unconfined flows with a knowledge of the jet width.

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Mixing Pressure-Rise Parameter for Effect of Nozzle Geometry in Diffuser-Ejectors

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Nomenclature

D = diameter
m = mass flow rate
p = static pressure
ϕ = pressure-rise parameter

Subscripts

p = primary
s = secondary

I. Introduction

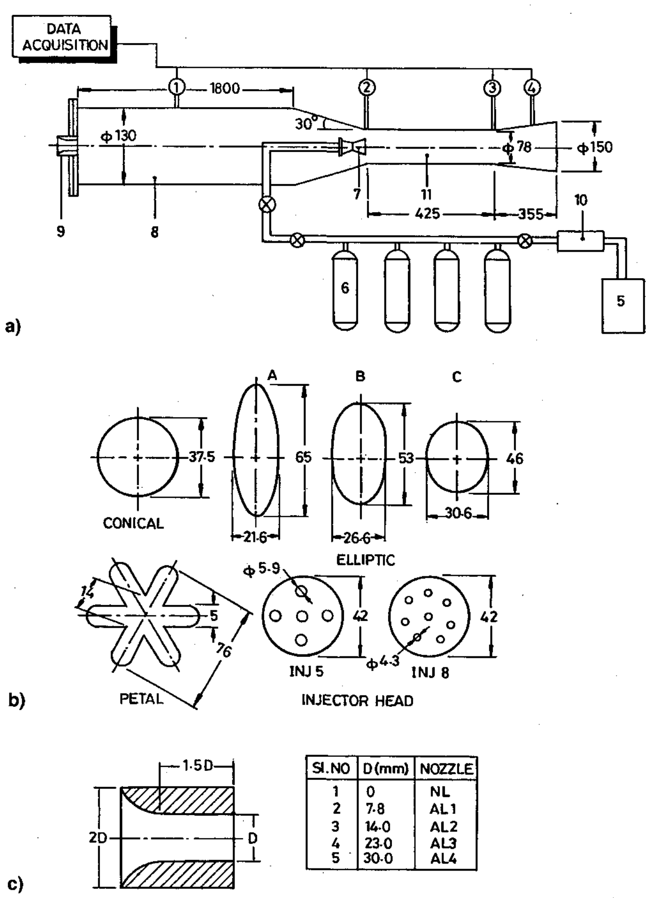
ONE of the technological areas in aerospace engineering where the ejector has proved indispensible is in high-altitude simulation.[1] Altitude testing requirements for space propulsion systems have led to the development of facilities that utilize the energy of the rocket exhaust itself in conjunction with ejector–diffuser systems to reduce the pressure in the test cell to values that simulate high-altitude conditions. Considerable theoretical and experimental work has been done with respect to the design and development of these systems.[2–8] However, most of these works have centered around circular cross-section primary nozzles. Little data seem to be available on the use of unconventional primary nozzles for ejector–diffuser systems. Therefore, it was considered appropriate to examine the performance of these systems using primary nozzles of various geometries. Results of these tests are presented in this Note.

II. Experimental Setup and Testing

The experimental facility used for these tests can be divided into the following subsystems: 1) air supply system, 2) test setup including nozzles, and 3) an instrumentation and data acquisition system.

A schematic diagram of the general test setup is shown in Fig. 1a. Compressed air was employed as the driving fluid for the ejector. The various primary nozzles to be tested were screwed onto the nozzle adapter. Pressure transducers of appropriate ranges were attached to the wall static ports. The transducer outputs were fed to the data acquisition system for real-time display and storage.

Seven primary nozzles of various geometries were tested (Fig. 1b). These include the conical nozzle (as a reference), three elliptic nozzles, two injector (shower) head types, and one petal[9] type of nozzle. The conical, elliptic, and petal nozzles were of the converging–diverging (C–D) type, whereas the two injector head nozzles were of the converging type. Exit area of the two latter nozzles was equal to the throat area of the other C–D nozzles. All of the C–D nozzles had circular



Sl.NO	D(mm)	NOZZLE
1	0	NL
2	7.8	AL1
3	14.0	AL2
4	23.0	AL3
5	30.0	AL4

Fig. 1 a) Experimental setup (1, 2, 3, 4 = wall static ports, 5 = compressor, 6 = storage tank, 7 = primary nozzle, 8 = secondary chamber, 9 = air-loading nozzle, 10 = dessicator, and 11 = mixing tube); b) primary; and c) air-loading nozzles.

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